Physics

B.Sc. - IInd year

Electromagnetic Theory
(B- 217)

Unit- I

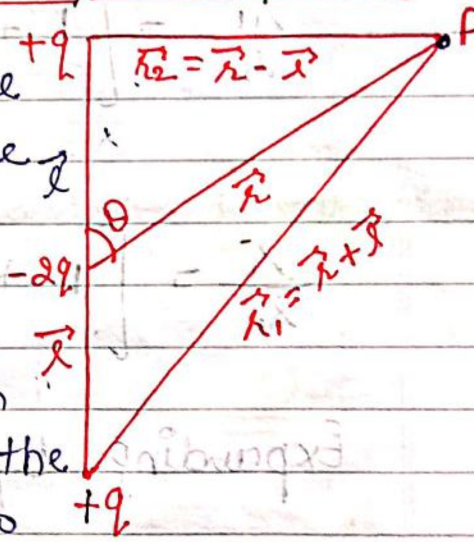
By - Dr. Vijay Kumar
(Physics Deptt.)
Deva Nagri College
Meerut

Electric Quadrupole

An electric quadrupole is that which consists of two equal and opposite dipoles that do not coincide in space so that their electric effects do not quite cancel each other at distant points.

Electric potential due to a Quadrupole

Let us consider a linear quadrupole with charge $-2q$ in the middle and charges $+q$, $+q$ at the ends.



Let P be distant point with position vector $\vec{r}$ at which the electrostatic potential is to find out.

The separation $\vec{l}$ b/w the charge is very small compared to the distance $\vec{r}$.

Let $\vec{r_1}$ and $\vec{r_2}$ be the distances of the point P from the charges $+q$, $+q$. Thus the potential at P due to the quadrupole is

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_1} + \frac{q}{r_2} - \frac{2q}{r}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r}\left[\frac{r}{r_1}+\frac{r}{r_2}-2\right] \quad \text{— (7)}$$

$$\vec{r_1} = |\vec{r}+\vec{l}|$$

$$= \left[(\vec{r}+\vec{l})\cdot(\vec{r}+\vec{l})\right]^{1/2}$$

$$= \left[r^2+l^2+2\vec{r}\cdot\vec{l}\right]^{1/2}$$

$$= r\left[1+\frac{l^2}{r^2}+\frac{2\vec{r}\cdot\vec{l}}{r^2}\right]^{1/2}$$

$$\frac{r_1}{r} = \left[1+\left\{\frac{l^2}{r^2}+\frac{2\vec{r}\cdot\vec{l}}{r^2}\right\}\right]^{1/2}$$

'OR'

$$\frac{r}{r_1} = \left[1+\left\{\frac{l^2}{r^2}+\frac{2\vec{r}\cdot\vec{l}}{r^2}\right\}\right]^{-1/2}$$

Expanding by binomial theorem, we get;

$$\frac{r}{r_1} = 1-\frac{1}{2}\left(\frac{l^2}{r^2}+\frac{2\vec{r}\cdot\vec{l}}{r^2}\right)+\frac{(-1/2)(-1/2-1)}{\underline{|2}}\left[\frac{l^2}{r^2}+\frac{2\vec{r}\cdot\vec{l}}{r^2}\right]^2$$

$$\because \vec{r}\cdot\vec{l} = rl\cos\theta \quad ----$$

$$\frac{r}{r_1} = 1-\frac{1}{2}\left[\frac{l^2}{r^2}+\frac{2l\cos\theta}{r}\right]+\frac{3}{8}\left[\frac{l^2}{r^2}+\frac{2l\cos\theta}{r}\right]^2 - ----$$

As $l <<< r$, neglecting the higher terms Then we get

$$\frac{r}{r_1} = 1 - \frac{1}{2}\frac{l^2}{r^2} - \frac{l}{r}\cos\theta + \frac{3}{8} \times \frac{4l^2\cos^2\theta}{r^2}$$

$$\frac{r}{r_1} = 1 - \frac{l}{r}\cos\theta + \frac{l^2}{2r^2}(3\cos^2\theta - 1)$$

Similarly;

$$\frac{r}{r_2} = 1 + \frac{l}{r}\cos\theta + \frac{l^2}{2r^2}(3\cos^2\theta - 1)$$

$$\therefore \frac{r}{r_1} + \frac{r}{r_2} = 2 + \frac{l^2}{r^2}(3\cos^2\theta - 1)$$

Put this value of $\frac{r}{r_1} + \frac{r}{r_2}$ in eq^n. (1) we get;

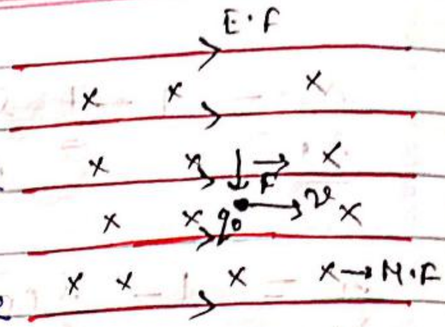
$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}\left[\cancel{2} + \frac{l^2}{r^2}(3\cos^2\theta - 1) - \cancel{2}\right]$$

$$\boxed{\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} q \frac{l^2}{r^3}(3\cos^2\theta - 1)}$$

This is the required expression.

★ Lorentz Force

The Lorentz force may be defined as "the force experienced by charged particle moving in a space where both electric and magnetic field exist. is known as Lorentz force."



Let us consider a test charge q_0 is moving in a space where both electric field $\vec{E}$ and magnetic field $\vec{B}$ are present. Then the force on a charge will be

$$\boxed{\vec{F} = q_0 \left[\vec{E} + \vec{v} \times \vec{B}\right]}$$

This force is called Lorentz force and this eqⁿ is called Lorentz force equation.

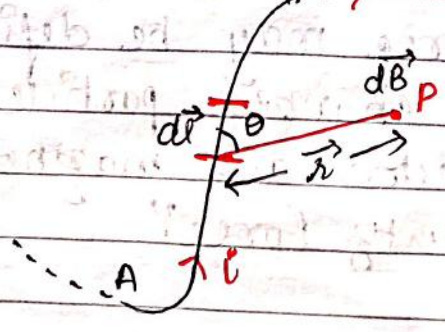
If $\vec{E} = 0$ then
$\vec{F} = q_0 (\vec{v} \times \vec{B})$

'OR'

$$\boxed{F = q_0 v B \sin\theta}$$

★ Biot - Savart's Law

A current carrying conductor produces a magnetic field around it. The magnitude and direction of this field at a point can be expressed by means of a law determined experimentally by Biot and Savart's and called as Biot-Savart's law.



Let 'i' be the current flow in the current carrying conductor AB. and 'P' be a point at which the field $(\vec{B})$ is to be find out. Let $d\vec{l}$ be the length of one such element. Let $\vec{r}$ be a position vector from the element to the point P. Then, according to Biot - Savart's law, the magnetic field induction $(d\vec{B})$ at point P due to the current element $d\vec{l}$ is given by.

$$d\vec{B} \propto \frac{i\, d\vec{l} \times \vec{r}}{r^3}$$

'OR'

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{l} \times \vec{r}}{r^3}$$

'OR'

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{i\, dl \sin\theta}{r^2}} \quad \text{——— (1)}$$

Where $\frac{\mu_0}{4\pi}$ is a proportionality constant and

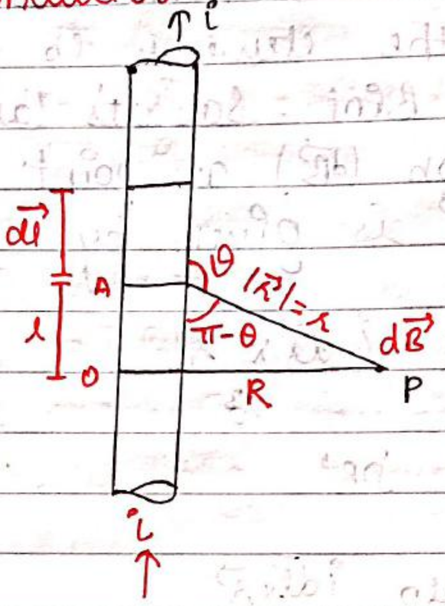
μ_0 is called permeability constant. Its value in S.I. system is $4\pi \times 10^{-7}$ Weber/Amperemeter.

This equation represents the Biot - Savart's law. Therefore, the resultant field induction due to the whole current carrying conductor at point P is given by

$$\boxed{B = \int dB}$$

Applications of Biot-Savart's law

1. Magnetic field due to a long, straight current carrying conductor



Let us consider an infinitely long conductor which is placed in a vacuum and carrying a current 'i' amp.

Let P be a point at which the magnetic field $d\vec{B}$ is to be find out

Here, OP = R and OA = l

Let $\vec{dl}$ is the current element at A. Then acc. to Biot-Savart's law the magnetic field induction $d\vec{B}$ at P due to current element $\vec{dl}$ is

$$d\vec{B} = \frac{\mu_0}{4\pi} i \frac{\vec{dl} \times \vec{r}}{r^3}$$

Its magnitude is given by

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{i\,dl \sin\theta}{r^2}} \quad \text{—— (1)}$$

In ΔOAP

$$\sin(\pi - \theta) = \frac{R}{r}$$

$$\sin\theta = \frac{R}{r}$$

$$r = \frac{R}{\sin\theta}$$

$$r = R\,\mathrm{cosec}\,\theta$$

Similarly;

$$\tan(\pi - \theta) = \frac{R}{l}$$

$$-\tan\theta = \frac{R}{l}$$

$$l = -R\cot\theta$$

Diff. above eqn. w.r.t 'θ' we get

$$\frac{dl}{d\theta} = R\,\mathrm{cosec}^2\theta$$

$$dl = R\,\mathrm{cosec}^2\theta \cdot d\theta$$

Putting the value of r and dl in eqn. ①, we get

$$dB = \frac{\mu_0}{4\pi} \frac{iR\,\mathrm{cosec}^2\theta\, d\theta \cdot \sin\theta}{R^2\,\mathrm{cosec}^2\theta}$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{i\sin\theta}{R}\, d\theta$$

Therefore, the magnitude of the magnetic field at P due to the whole conductor is

$$B = \int dB$$

$$B = \int \frac{\mu_0}{4\pi} \frac{i\sin\theta}{R}\, d\theta$$

$$= \frac{\mu_0}{4\pi} \frac{i}{R} \left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{\mu_0}{4\pi} \frac{i}{R} \left[-\cos\pi + \cos 0\right]$$

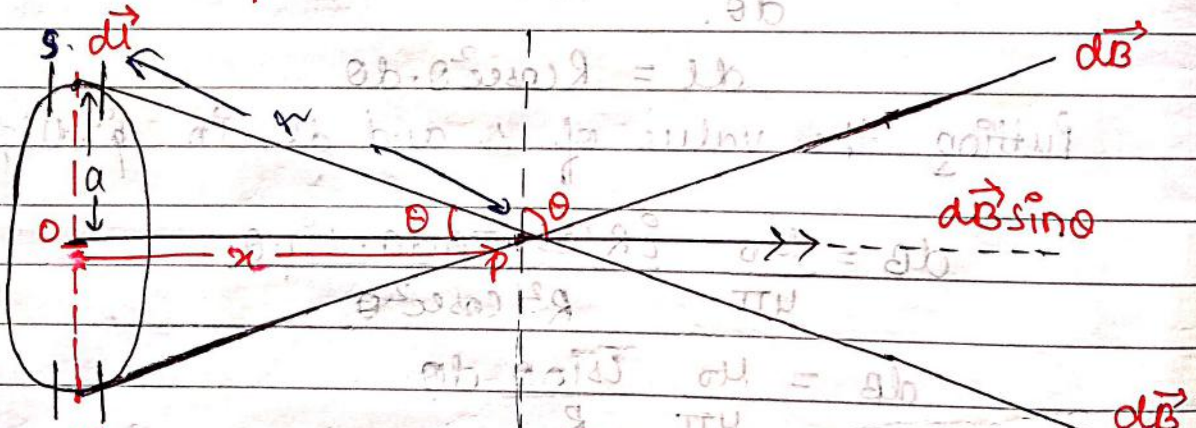
$$= \frac{\mu_0}{4\pi} \frac{i}{R} \left[-(-1)+1\right]$$

$$\boxed{B = \frac{\mu_0 i}{2\pi R}}$$

This is the expression for the magnetic field intensity due to a long straight conductor.

2. Magnetic field along the axis of a circular current loop

I.Q.



Let there be a circular coil of radius 'a' carrying a current 'i'. Let P be a point on the axis of the coil, distance 'x' from the centre at which the field is to be find out.

Let $\vec{dl}$ be a current-element at the top of the coil. If $\vec{r}$ be the displacement vector from the current-element to 'P'. Then from Biot-Savart's law, the magnetic field induction at P due to the current-element $\vec{dl}$ is given by

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{i\,\vec{dl} \times \vec{r}}{r^3}$$

OR,

$$dB = \frac{\mu_0}{4\pi} \frac{i\,dl \sin\theta}{r^2}$$

Because the angle b/w $\vec{dl}$ and $\vec{r}$ is $90°$. Then

$$dB = \frac{\mu_0}{4\pi} \frac{i\,dl}{r^2} \quad \text{——— (1)}$$

$\vec{dl}$ and $\vec{r}$ both are $\perp r$ to the direction of $\vec{dB}$. The vertical components of the magnetic field $dB \cos\theta$ is cancelled due to the equal and opposite direction. But the horizontal components are effective.

Thus the resultant field $\vec{B}$ at P due to the complete loop is given by

$$B = \int dB \sin\theta$$

Put the value of dB in above eq^n. from eq^n. (1) we get;

$$B = \int \frac{\mu_0}{4\pi} \frac{i\,dl \sin\theta}{r^2}$$

$$B = \frac{\mu_0}{4\pi} \frac{i}{r^2} \sin\theta \int dl$$

$$\because \int dl = 2\pi a$$

$$\therefore B = \frac{\mu_0}{\cancel{4\pi}_{2}} \frac{i}{r^2} \sin\theta \times 2\cancel{\pi} a$$

$$B = \frac{\mu_0 i a \sin\theta}{2r^2}$$

$$\therefore B = \frac{\mu_0 i a \times \frac{a}{r}}{2r^2}$$

$\left[\because \text{In } \Delta OPS \quad \sin\theta = \frac{a}{r} \right.$

$$B = \frac{\mu_0 i a^2}{2r^3} \quad \text{--- (2)}$$

In ΔOPS;

$$r^2 = a^2 + x^2$$

$$r = (a^2 + x^2)^{1/2}$$

"OR"

$$r^3 = (a^2 + x^2)^{3/2}$$

Put the value of r^3 in eqn. (2)

$$\boxed{B = \frac{\mu_0 i a^2}{2(a^2 + x^2)^{3/2}}}$$

Let the no. of turns N in the coil. Then, the magnetic field induction will be

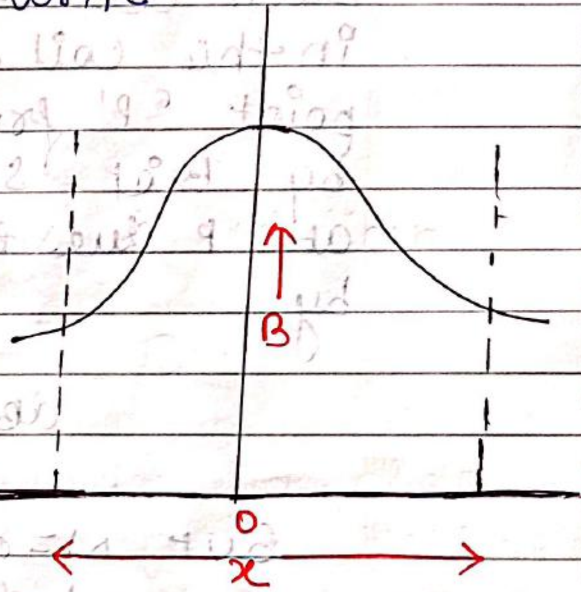
$$\boxed{B = \frac{\mu_0 N i a^2}{2(a^2+x^2)^{3/2}}} \quad \text{--- (3)}$$

It is clear from the above eqn. that the value of B depend on the value of x. Thus if we consider the point 'P' at the centre of coil i.e; x = 0

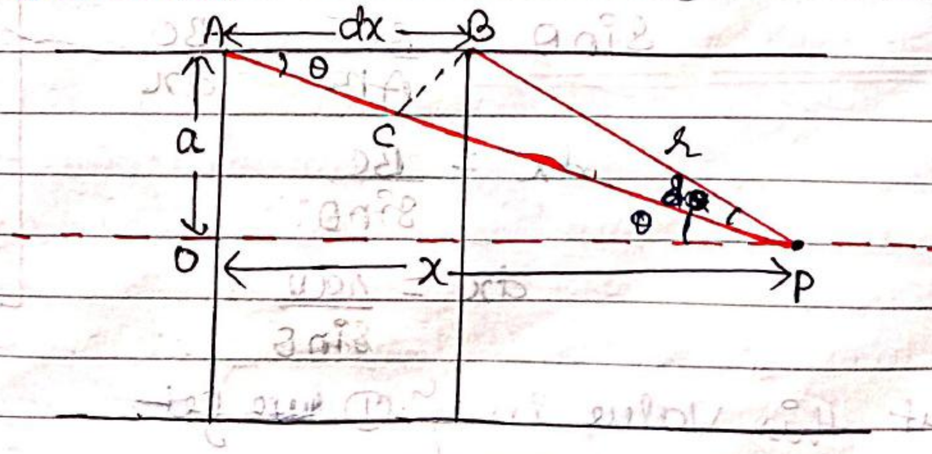
Hence from eqn. (3) we may write

$$\boxed{B = \frac{\mu_0 N i}{2a}}$$

This is the maximum value of magnetic field.



3. To find the magnetic field due to a long current carrying solenoid



Let us consider a long solenoid of radius 'a' meter and carrying a current of 'i' amp. Let 'n' be the no. of turns per meter length of the solenoid. Let P be a point on the axis of the solenoid at which the field is to be find out.

Let us imagine the solenoid to be divided up into a no. of narrow coils and consider one such coil AB of width dx. The no. of turns in this coil is ndx. Let x be the distance of the point 'P' from the centre 'O' of this coil. Then, by Biot - Savart's law, the magnetic induction at P due to this elementary coil is given by

$$dB = \frac{\mu_0 N i a^2}{2(a^2+x^2)^{3/2}}$$

But $N = ndx$

$$\therefore dB = \frac{\mu_0 n i a^2}{2(a^2+x^2)^{3/2}} dx \quad \text{——— (1)}$$

In ΔABC

$$\sin\theta = \frac{BC}{AB} = \frac{BC}{dx}$$

$$dx = \frac{BC}{\sin\theta}$$

$$dx = \frac{r d\theta}{\sin\theta}$$

$\left[\because \angle le = \frac{arc}{radius} \quad d\theta = \frac{BC}{r} \quad BC = r d\theta \right.$

Put this value in eq^n. (1) we get

$$dB = \frac{\mu_0 n i a^2}{2(a^2+x^2)^{3/2}} \cdot \frac{r d\theta}{\sin\theta}$$

$\left[\text{In } \Delta PAO \quad r^2 = (a^2+x^2) \right.$

mycompanion

$$\therefore dB = \frac{\mu_0 n i a^2}{2(r^2)^{3/2}} \cdot \frac{r\, d\theta}{\sin\theta}$$

$$= \frac{\mu_0 n i a^2}{2r^2} \frac{d\theta}{\sin\theta}$$

$$= \frac{\mu_0 n i}{2} \left[\frac{a}{r}\right]^2 \frac{d\theta}{\sin\theta}$$

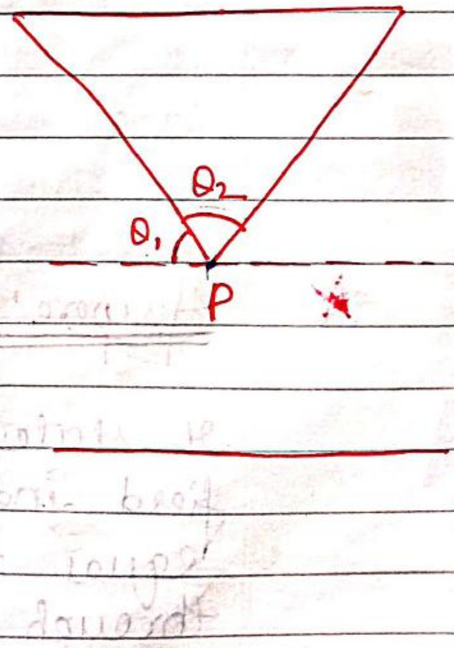
But from figure

$$\frac{a}{r} = \sin\theta$$

$$\therefore dB = \frac{\mu_0 n i}{2} \sin^2\theta \frac{d\theta}{\sin\theta}$$

$$dB = \frac{1}{2} \mu_0 n i \sin\theta \, d\theta$$

Then the field induction B at P due to the whole solenoid is given by



$$B = \int dB$$

$$= \frac{1}{2} \mu_0 n i \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= \frac{1}{2} \mu_0 n i \left(-\cos\theta\right)_{\theta_1}^{\theta_2}$$

$$\boxed{B = \frac{1}{2} \mu_0 n i \left(\cos\theta_1 - \cos\theta_2\right)}$$

Case: 1 When the length of solenoid is very long, then $\theta_1 = 0$, $\theta_2 = \pi$

$$\therefore B = \frac{1}{2}\mu_0 n i\,(\cos 0 - \cos \pi)$$

$$= \frac{1}{2}\mu n i\,(1+1)$$

$$\boxed{B = \mu_0 n i}$$

<u>Case: II</u> A the end of the solenoid

$\theta_1 = 0, \quad \theta_2 = 90°$

$$B = \frac{1}{2}\mu_0 n i\,(\cos 0 - \cos 90°)$$

$$\boxed{B = \frac{1}{2}\mu_0 n i}$$

<u>Case: III</u> At the first end of the solenoid

$\theta_1 = 90°, \quad \theta_2 = 180°$

$$\boxed{B = \frac{1}{2}\mu_0 n i}$$

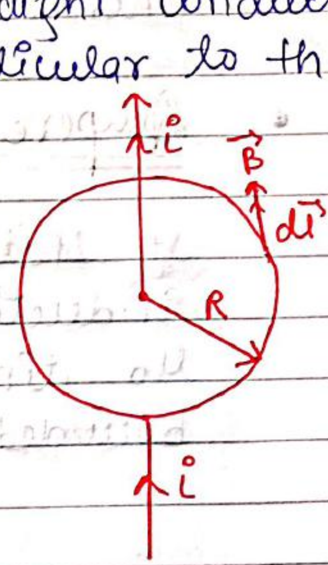
★ Ampere's Circuital Law

It states that the line integral of the magnetic field induction $\vec{B}$ around a closed curve is equal to the μ_0 times the net current 'i' through the area bounded by the curve i.e;

$$\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 i}$$

Where μ_0 is the permeability of free space.

Proof: Let us consider a long, straight conductor carrying a current 'i' perpendicular to the page direction outward. Acc. to Biot - Savarts law, the magnitude of the magnetic induction ~~of the magnetic~~ at a distance R from it is given by



$$B = \frac{\mu_0}{2\pi}\frac{i}{R} \text{ ——— (i)}$$

and its direction is tangent to a circle of radius R centered on the wire.
Let us imagine the circle is made up of many no. of current carrying element and consider one such element $\vec{dl}$ and its direction will be along $\vec{B}$. Then we have the line integral

$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl$$

$$= B\oint dl$$

$$\oint \vec{B}\cdot\vec{dl} = B \times 2\pi R \text{ ——— (2)}$$

Putting the value of B from eqn. (1) in eqn. (2) we get

$$\oint \vec{B}\cdot\vec{dl} = \frac{\mu_0}{\cancel{2\pi R}}\, i \times \cancel{2\pi R}$$

$$\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 i}$$

This is Ampere's law.

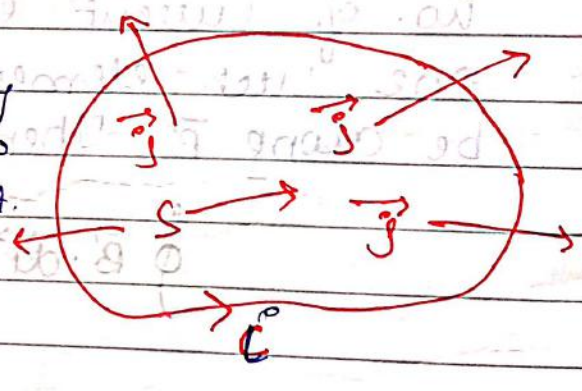
- **Ampere's law in differential form**

It states that the line integral of the magnetic induction around a closed curve is equal to μ_0 times the net current across the ~~[illegible]~~ area bounded by the curve i.e;

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i \quad \text{——} \quad (1)$$

This is Ampere's law in circuital 'or' integral form.

Let us consider a closed curve 'c' in a region where current is flowing.
Let $\vec{j}$ be the current density which varies from place to place, but it is constant in time. Then the total current is



$$i = \int_S \vec{j} \cdot \vec{ds}$$

Put this value in eqn. (1) we get

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \int_S \vec{j} \cdot \vec{ds} \quad \text{——} \quad (2)$$

From the Stoke's theorem, we have

$$\oint \vec{B} \cdot \vec{dl} = \int_S \text{curl}\, \vec{B} \cdot \vec{dS} \quad \text{——} \quad (3)$$

from eqⁿ. (2) and eqⁿ. (3) we may write

$$\text{Curl } \vec{B} = \mu_0 j$$

'OR'

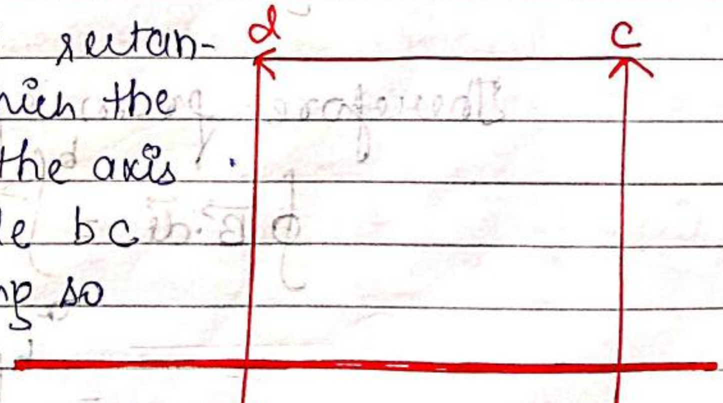
$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 j}$$

This is the required expression for the Ampere's law in differential form.

Magnetic field of a solenoid

Let us consider a solenoid very long compared with its diameter and carrying a current 'i'. We experimentally observe that for such a solenoid the magnetic field outside is very small compared with the field inside.

Let us consider a closed rectangular path abcd in which the side ab is ||el to the axis of the solenoid and side bc and da are very long so that the side cd is far from the solenoid and the field at this side when the solenoid is long and the rectangle is not too near either end, the field is at right angles to the sides bc and da.



Let the length of ab is 'x'. Then by the Ampere's law

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i \quad \text{--- (1)}$$

Applying this law to the rectangular path abcd, we get

$$\oint \vec{B}\cdot\vec{dl} = \int_a^b \vec{B}\cdot\vec{dl} + \int_b^c \vec{B}\cdot\vec{dl} + \int_c^d \vec{B}\cdot\vec{dl} + \int_d^a \vec{B}\cdot\vec{dl} \quad \text{--- (2)}$$

But

$$\int_b^c \vec{B}\cdot\vec{dl} = \int_a^d \vec{B}\cdot\vec{dl} = 0$$

{Because along bc and da the field $\vec{B}$ is at right angles to $\vec{dl}$

And

$$\int_c^d \vec{B}\cdot\vec{dl} = 0$$

{Because $\vec{B}$ is zero at points outside the solenoid

Therefore from eq^n. (2), we may write

$$\oint \vec{B}\cdot\vec{dl} = \int_a^b \vec{B}\cdot\vec{dl}$$

$$= \int_a^b B\,dl\cos\theta$$

$$\therefore \oint \vec{B}\cdot\vec{dl} = \int_a^b B\,dl \qquad [\because \theta = 0^\circ]$$

$$= B\int dl$$

$$\oint \vec{B}\cdot\vec{dl} = Bx \quad \text{------} \quad (3)$$

Let 'n' be the no. of turns per unit length of the solenoid then no. of turns in length ab will be nx.

Now, the current through each turn is 'i_0'. Therefore, the amount of current enclosed in the rectangle will be

$$i = i_0 nx \quad \text{------} \quad (4)$$

Putting the value of $\oint \vec{B}\cdot\vec{dl}$ and 'i' in eq^n. (1) we get

$$Bx = \mu_0 n x i_0$$

$$\boxed{B = \mu_0 n i_0}$$

Thus it is clear from the above expression that the field independent of the length and dia-meter of the solenoid.

Numericals :-

Q.1: A copper wire 0.254 cm in diameter carries a current of 50 amp. Find the magnetic field induction B at the surface of the wire. The permeability constant μ_0 is $4\pi \times 10^{-7}$ weber/amp-m

Sol^n Diameter = 0.254 cm

$2R = 0.254$

$R = 0.127$ cm

$R = 0.127 \times 10^{-2}$ m

$i = 50$ amp.

$\mu_0 = 4\pi \times 10^{-2}$ webes/amp-m

At the surface of the wire

$$B = \frac{\mu_0 i}{2\pi R}$$

$$= \frac{4\pi \times 10^{-7} \times 50}{2 \times \pi \times 0.127 \times 10^{-2}}$$

$$= \frac{1000}{127} \times 10^{-3}$$

$$= 7.87 \times 10^{-3}$$

$$\boxed{B = 7.87 \times 10^{-3} \text{ webes/m}^2}$$ Ans.

Ques. 2. A long straight vertical wire carries a current of 30 amp. directed upward. Calculate the position of neutral point if the horizontal component of earth's magnetic field is 2×10^{-5} wb/m^2. given $\mu_0 = 4\pi \times 10^{-7}$ wb/Amp-m.

Sol^n $i = 30$ amp.

$B = 2 \times 10^{-5}$ web/m^2

$\mu_0 = 4\pi \times 10^{-7}$ web/amp.m

$$B = \frac{\mu_0 i}{2\pi R}$$

$$2 \times 10^{-5} = \frac{4\pi \times 10^{-7} \times 30}{2\pi R}$$

$$2 \times 10^{-5} \times R = 2 \times 3 \times 10^{-6}$$

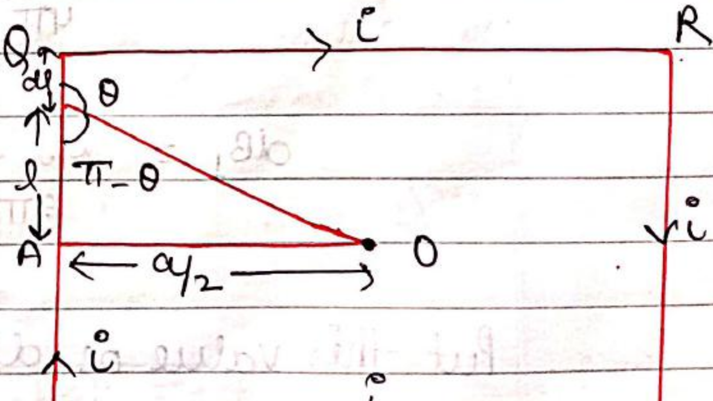
$$R = 3 \times 10^{-1} m$$

$$R = 0.3 m$$

OR

$$\boxed{R = 30 cm}$$

Ques 3. A square loop of wire of side 'a' carries a current 'i'. Show that the value of B at the centre is given by $B = \frac{2\sqrt{2}\mu_0 i}{\pi a}$.



Soln. Let PQRS be a square current loop of side a. Let O be the centre of loop at which the field is required.

Let $d\vec{l}$ be a current element in the side PQ at a distance 'l' from A. If $\vec{r}$ be the radius vector from the element to O. Then by Biot-Savart's law the field $d\vec{B}$ at 'O' due to the magnetic field.

$$B_1 = \int dB_1 \quad \text{——— (A)}$$

The magnetic flux at 'O' due to the element $d\vec{l}$ is

$$dB_1 = \frac{\mu_0}{4\pi} \frac{i\,dl\sin\theta}{r^2} \quad \text{——— (1)}$$

from figure, $\sin(\pi - \theta) = \frac{a/2}{r}$

$$\therefore \sin\theta = \frac{a}{2r}$$

And

$$r^2 = l^2 + \frac{a^2}{4}$$

Put these value of r^2 and $\sin\theta$ in eq^n. (1) we get

$$dB_1 = \frac{\mu_0}{4\pi} \frac{i\,dl\,a/2}{r \cdot r^2}$$

$$dB_1 = \frac{\mu_0}{4\pi} \times \frac{i\,dl\,a}{r^3}$$

$$dB_1 = \frac{\mu_0 i a}{8\pi} \times \frac{dl}{\left[l^2 + \frac{a^2}{4}\right]^{3/2}}$$

Put this value of dB_1 in eq^n. (A) we get

$$B_1 = \int dB_1$$

$$B = \frac{\mu_0 i a}{8\pi} \int_{-a/2}^{a/2} \frac{dl}{\left(l^2 + \frac{a^2}{4}\right)^{3/2}}$$

Let $l = \frac{a}{2}\tan\phi$

diff. w.r.t 'ϕ'

$$\frac{dl}{d\phi} = \frac{a}{2}\sec^2\phi$$

$$dl = \frac{a}{2}\sec^2\phi \, d\phi$$

$$\therefore \quad B_1 = \frac{\mu_0 i a}{8\pi}\int_{-\pi/4}^{\pi/4} \frac{a/2 \sec^2\phi \cdot d\phi}{\left[\frac{a^2}{4}\tan^2\phi + \frac{a^2}{4}\right]^{3/2}}$$

$$\because \quad l = \frac{a}{2}\tan\phi$$

$$\frac{a}{2} = \frac{a}{2}\tan\phi$$

$$\tan\phi = 1 = \tan\frac{\pi}{4}$$

$$\boxed{\phi = \frac{\pi}{4}}$$

$$B_1 = \frac{\mu_0 i a}{8\pi} \cdot \frac{a}{2}\int_{-\pi/4}^{\pi/4} \frac{\sec^2\phi}{(a^2/4)^{3/2}(\tan^2\phi + 1)^{3/2}}$$

$$= \frac{\mu_0 i a}{8\pi} \cdot \frac{a}{2}\int_{-\pi/4}^{\pi/4} \frac{\sec^2\phi}{a^3/8 \sec^3\phi} d\phi$$

$$= \frac{\mu_0 i a}{8\pi} \cdot \frac{a}{2}\int_{-\pi/4}^{\pi/4} \frac{8 \, d\phi}{a^3 \sec^2\phi}$$

$$= \frac{\mu_0 i a}{8\pi} \cdot \frac{a}{2} \cdot \frac{8}{a^3} \int_{-\pi/4}^{\pi/4} \cos\phi \, d\phi$$

$$= \frac{\mu_0 i}{2\pi a} \left[\sin\theta \right]_{-\pi/4}^{\pi/4}$$

$$B_1 = \frac{\mu_0 i}{2\pi a} \left[\sin\frac{\pi}{4} - \sin\left(\frac{-\pi}{4}\right) \right]$$

$$B_1 = \frac{\mu_0 i}{2\pi a} \left[\sin\frac{\pi}{4} + \sin\frac{\pi}{4} \right]$$

$$B_1 = \frac{\mu_0 i}{\pi a \sqrt{2}}$$

$$B_1 = \frac{\sqrt{2} \mu_0 i}{2\pi a}$$

Therefore, the field $\vec{B}$ due to the entire loop will be four times the field due to one arm i.e.

$$B = 4B_1$$

$$= \frac{4 \mu_0 i}{\sqrt{2} \pi a}$$

$$\boxed{B = \frac{2\sqrt{2} \mu_0 i}{\pi a}}$$

Ques. 4. A long wire having a semi-circular loop of radius 'r' carries a current 'i'. Find the magnetic induction at the centre c